Pre-class Warm-up!!!

How many ways can you think of to describe a curve in R^2 ?

a. ≥ 4

b. 3

c. 2

d. 1

e. 0

The quiz tomorrow is on the material from both last week and the week before. I think it was sections 8.3 6.1 6.2 1.4 but I don't remember.

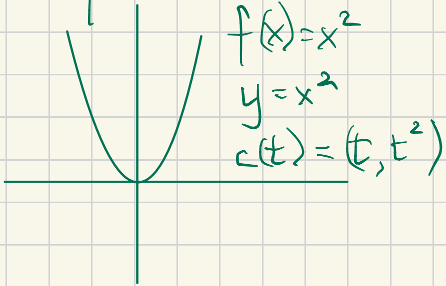
7.3 Parametrized Surfaces.

We learn:

- what is a surface. It is a sheet that may bend
- we have 3 ways to describe a surface
- How to compute the tangent plane at a point from a parametrization.

- graph of $f(x,y)$
- points satisfying an equation like $z-x^2+y^2=0$

  We can compute tangent planes from this using the gradient
- parametrizations — today!



Review: we can sometimes describe a curve in 3 ways:

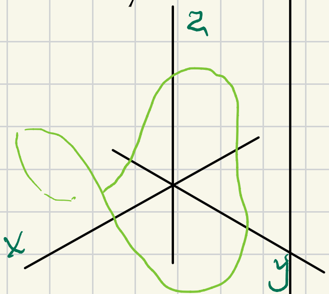
- the graph of a function, like the graph of f(x) = x^2
- The solution set of some equations, like y = x^2
- by a parametrization, like c(t) = (t, t^2)

We know how to convert between these and how to compute tangent lines to curves.

Definition. A parametrization of a surface in R^3 is a function $\Phi: \mathbb{R}^2 \longrightarrow \mathbb{R}^3$

$$\Phi(u,v) = (x(u,v), y(u,v), z(u,v))$$

Examples: The graph of f(x,y) = x^2 - y^2



- has a parametrization

$$\Phi(u,v) = (u, v, u^2 - v^2)$$

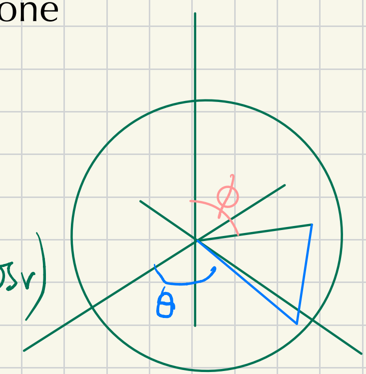
Φ is the parametrization

The image of Φ is the surface.

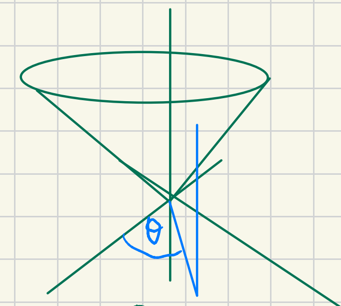
- is the set of points satisfying an equation

$$z = x^2 - y^2 .$$

- We should recognize parametrizations of a unit sphere, and of a cone



$$\Phi(u,v) = (\sin v \cos u, \sin v \sin u, \cos v)$$



$$\Phi(u,v) = (u \cos v, u \sin v, u)$$

Think $u = r$, $u \geq 0$, $v = \theta$, $0 \leq v < 2\pi$

Tangent vectors

Recall: for a path c : R -> R^3 the velocity vector c′ points tangentially to the curve.

$$c'(t) = \left(\frac{dx}{dt}, \frac{dy}{dt}, \frac{dz}{dt}\right)$$

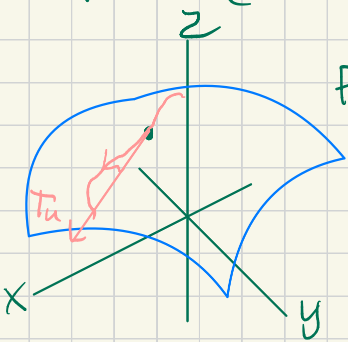
Given a parametrization of a surface

$$\Phi(u,v) = (x(u,v), y(u,v), z(u,v))$$

we get two vectors tangential to the surface:

$$T_u = \left(\frac{\partial x}{\partial u}, \frac{\partial y}{\partial u}, \frac{\partial z}{\partial u}\right)$$

$$T_v = \left(\frac{\partial x}{\partial v}, \frac{\partial y}{\partial v}, \frac{\partial z}{\partial v}\right)$$



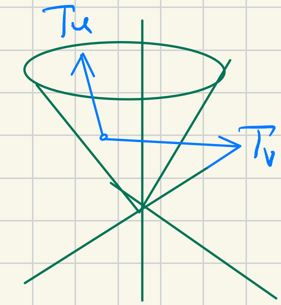
Fix v and let u vary to get a path in the surface.

Example: the parametrization of the cone:

Phi (u,v) = (u cos(v), u sin(v), u),
$u \geq 0$, $0 \leq v \leq 2\pi$

$$T_u = (\cos v, \sin v, 1)$$

$$T_v = (-u\sin v, u\cos v, 0)$$



We get a normal vector

$$T_u \times T_v = (-u\cos v, -u\sin v, u)$$

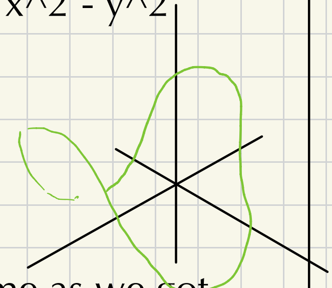
we construct the tangent plane

$$-u\cos v\,(x - u\cos(v)) + \;.\; + \;\ldots\; = 0$$

We always want the parametrization to be regular, meaning $T_u \times T_v \neq 0$

Example: the graph of f(x,y) = x^2 - y^2 has parametrization

Phi (u,v) = (u, v, u^2 - v^2)



The tangent vectors are the same as we got previously.

$T_u = (1, 0, 2u)$, $T_v = (0, 1, -2v)$

Question: are either of the following surfaces (that we just saw) the graphs of functions?

a. the sphere Yes No

b. the cone Yes No